Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
In Pure Mathematics P2 (WMA12) Paper 01

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January 2022
Question Paper Log Number P66646A
Publications Code WMA12_01_2201_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\lfloor$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the ‘0’ column when it was meant to be ‘1’ and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \ldots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \ldots$$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme	Notes	Marks
1(a)	$h = 0.5$	Correct strip width	B1
	$A \approx \frac{1}{2}\times\frac{1}{2}\{6.792 + 5.113 + 2(6.298 + 5.858 + 5.466)\}$ Correct application of the trapezium rule with their *h*		M1
	= 11.79	Cao	A1
			(3)
(b)(i)	$A \approx 2\times"11.79"$	Multiplies their answer to (a) by 2	M1
	= 23.58	Correct answer or correct ft	A1ft
(b)(ii)	$A \approx "11.79" + 6$	Adds 6 to their answer to (a)	M1
	= 17.79	Correct answer or correct ft	A1ft
			(4)
			Total 7

Notes:
(a)

B1: Correct value for *h* stated or implied by $\frac{1}{4}\{...\}$

M1: Attempts the trapezium rule with their *h*. Must include the $\frac{1}{2}\times"h"$ but this may be implied by any multiple of the bracket if *h* is not identified separately. Must have the correct inner bracket structure, though allow if all that is wrong is a miscopy of numerals. May be written as separate trapezia.

Bracketing error $\frac{1}{4}(6.792 + 5.113) + 2(6.298 + 5.858 + 5.466)$ is M0 unless recovered by their answer.

A1: cao 11.79. Must be to 2 d.p. (Note the actual value is 11.78 to 2 d.p. and score no marks if just this is seen)

(b)(i)

M1: For twice their answer to (a), allowing for rounding. No need to carry out the doubling, sight of 2 $2\times"\text{their }(a)"$ is sufficient.

A1ft: For 23.58 or awrt 23.57 following a correct part (a) rounded or truncated to 11.78 (if they use more than 2d.p. then 23.5745 is the value they should get) **or** follow through their answer to (a) using the same principle. May be given to more than 2 d.p. and isw. (Allow answer to less than 2d.p. if just trailing zeros are omitted.)

Note: accurate answer is 23.56 to 2 d.p., which is M0A0 if no method is shown.

(ii)

M1: For 6+ their (a) stated or implied (but see note). The 6 need not be simplified if evaluated from an integral (but the integral and substitution must be correct giving an evaluation that simplifies to 6).

A1ft: 17.79 or follow through their answer to (a) + 6 evaluated. May be given to more than 2 d.p. and isw

Note: accurate answer is 17.78 to 2 d.p., which is M0A0 if no method is shown.

Note: Repeated trapezium rule is M0 in (b).

Question Number	Scheme	Notes	Marks
2(a)	$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20$	$x^n \to x^{n-1}$	M1
	$\Rightarrow \left(\dfrac{dy}{dx} =\right) \dfrac{27}{2}x^{-\frac{1}{2}} - \dfrac{3}{2}x^{\frac{1}{2}}$	Correct simplified derivative	A1
			(2)
(b)	$\dfrac{27}{2}x^{-\frac{1}{2}} - \dfrac{3}{2}x^{\frac{1}{2}} = 0 \Rightarrow x = \ldots$	Sets their derivative = 0 attempts to solve for *x*	dM1
	$x = 9$	Correct *x* value	A1
	$x = 9 \Rightarrow y = \ldots$	Uses their *x* value to find a value for *y*	M1
	$y = 34$	Correct *y* value	A1
			(4)
(c)	$\dfrac{d^2y}{dx^2} = \pm Ax^{-\frac{3}{2}} \pm Bx^{-\frac{1}{2}}$ $\left(\dfrac{d^2y}{dx^2}\right)_{x=9} = -\dfrac{27}{4}(9)^{-\frac{3}{2}} - \dfrac{3}{4}(9)^{-\frac{1}{2}}\left(= -\dfrac{1}{2}\right)$	Attempts second derivative and substitutes their *x* **or** considers the sign (for *x* >0)	M1
	$\dfrac{d^2y}{dx^2} = -\dfrac{27}{4}x^{-\frac{3}{2}} - \dfrac{3}{4}x^{-\frac{1}{2}}$ $\dfrac{d^2y}{dx^2} < 0$ for x > 0 / when *x* = 9 so maximum	Correct second derivative and conclusion with correct reason	A1
			(2)
			Total 8

Notes

(a)

M1: Power decreased by 1 in at least one term in *x*

A1: Correct, simplified derivative. Accept as shown or $\dfrac{27}{2\sqrt{x}} - \dfrac{3}{2}\sqrt{x}$ or with decimal equivalents for coefficients. No need to see the $\dfrac{dy}{dx}$, score for a correct expression.

(b)

Note: Answer only with no initial equation set up scores no marks.

dM1: **Depends on the M in (a).** Sets their derivative equal to 0 and reaches a value *x*. The "=0" may be implied by a clear attempt to solve it. Do not be concerned about the method for solving as long as a value of *x* is reached. If substituting $a = \sqrt{x}$ (or similar) must return to *x* for the method.

A1: Correct value for *x* from correct work. Ignore references to *x* = 0 or any negative values but A0 if extra positive values.

M1: Substitutes their positive *x* value into the curve equation to find a value for *y*. May need to check if no substitution is shown.

A1: Correct *y* value. Ignore reference to any point at *x* = 0 or negative values but A0 if other coordinates with positive *x* are given.

(c) Ignore work relating to *x* = 0 in (c)

M1: Attempts the second derivative achieving the form $\pm Ax^{-\frac{3}{2}} \pm Bx^{-\frac{1}{2}}$ and attempts to use it to classify the stationary point. Look for an attempt at substituting their value or a consideration of the sign.

A1: Correct second derivative and conclusion drawn with supporting evidence from which it is reasonable to deduce the nature. Accept "concave down" for maximum, but not just "concave".

For the evidence accept either evaluation to the correct value $-\frac{1}{2}$, or correct sign deduced from substitution of $x = 9$, or via **statement** it is negative for all $x > 0$ without substitution seen. Use of a value other than $x = 9$ scores A0.

Question Number	Scheme	Notes	Marks
3(a)	$\left(2-\frac{kx}{4}\right)^8 = 2^8 + \binom{8}{1}2^7\left(-\frac{kx}{4}\right) + \binom{8}{2}2^6\left(-\frac{kx}{4}\right)^2 + \binom{8}{3}2^5\left(-\frac{kx}{4}\right)^3 + \ldots$ Or $\left(1-\frac{kx}{8}\right)^8 = 1 + \binom{8}{1}\left(-\frac{kx}{8}\right) + \binom{8}{2}\left(-\frac{kx}{8}\right)^2 + \binom{8}{3}\left(-\frac{kx}{8}\right)^3 + \ldots$		M1
	$= 256 - 256kx + 112k^2x^2 - 28k^3x^3 + \ldots$	$256 - 256kx$	B1
		$112k^2x^2$ **or** $-28k^3x^3$ (unsimplified)	A1
		$112k^2x^2$ **and** $-28k^3x^3$ (simplified)	A1
			(4)
(b)	$f(x) = (5-3x)\left(2-\frac{kx}{4}\right)^8 = (5-3x)\left(256 - 256kx + 112k^2x^2 - 28k^3x^3 + \ldots\right)$ Coefficient of x is $5\times -256k - 3\times 256$		M1
	$5\times 256 = 3(-1280k - 768) \Rightarrow k = \ldots$	Sets $5\times$their constant term from (a) = $3\times$their coefficient of x from f(x) and solves for k	M1
	$k = -\frac{14}{15}$	Correct value.	A1
			(3)
			Total 7

Notes

(a)

M1: Attempts the binomial expansion on $\left(2 \pm \frac{kx}{4}\right)^8$ or $\left(1 \pm \frac{kx}{\beta}\right)^8$ up to at least the third (x^2) term with an acceptable structure. Look for the correct binomial coefficient (accept alternative notation ${}^nC_r$) combined with the correct power of x but allow if powers of 2 are incorrect and if brackets are missing. M0 for descending powers.

B1: for $256 - 256kx$, may be listed, must be simplified. Allow for $256(1 - kx + \ldots)$ if the 2^8 is taken out first.

A1: Correct third or fourth term, may be listed. Need not be simplified but the binomial coefficients must be numerical. Allow for one term from $256\left(\ldots + \frac{28}{64}(kx)^2 - \frac{56}{512}(kx)^3 \ldots\right)$ if the 2^8 is taken out first. May have powers as $(kx)^n$ for this mark. Allow for the correct x^2 term if the sign was incorrect in their bracket.

A1: Correct simplified third and fourth terms as shown in scheme, may be listed. Must have k^nx^n terms.

Note: isw after correct terms are seen if they try to divide through.

(b)

M1: Correct strategy for the coefficient of x or the x term. E.g. $5\times$their $-256k$ – $3\times$their 256 or may be part of a full expansion – look for ($5\times$their $-256k$ – $3\times$their 256)x but terms must have been combined.

M1: Sets $5\times$their constant term from (a) = $3\times$their coefficient of x from f(x) and solves for k. Should be an equation in k only, but allow recovery if they initially include the x but later cross it out to give a constant for the answer. The attempt at the x coefficient must have been an attempt at a sum of two terms from their expansion of f(x)

A1: Correct value, must be exact. Allow $-0.9\dot{3}$ but not a terminating decimal.

Question Number	Scheme	Notes	Marks
4	$2\log_3(1-x)=\log_3(1-x)^2$ or $3=\log_3 3^3$	Correct power law used or implied	B1
	$\log_3(32-12x)-\log_3(1-x)^2=\log_3\dfrac{32-12x}{(1-x)^2}$ Combines 2 log terms correctly		M1
	$\dfrac{32-12x}{(1-x)^2}=27$	Obtains this equation in any form	A1
	$\Rightarrow 27x^2-42x-5=0\Rightarrow x=...$	Solves 3TQ	M1
	$x=-\dfrac{1}{9}$	This value only i.e. the $\dfrac{5}{3}$ must clearly be discarded if seen.	A1
			(5)
			Total 5

Notes

(a)

B1: Correct use of implication of the power law. Award for sight any of $\log_3(1-x)^2$, $\log_3 3^3$, $\log_3 27$ or for $3\to 27$ moving from $\log_3(...)=3$ or ...= 27 (oe)

Allow for $\log_3\left((1-x)+3\right)^2$ as a misread (the M will be lost)

Allow B1 for $2\log_3(1-x)+3\to\log_3 3(1-x)^2$ without first seeing the $\log_3(1-x)^2$ but again the M will be lost without further sufficient work seen.

M1: For correctly combining two of the terms $\log_3(32\pm12x),\log_3(1\pm x)^2$ or $\log_3 A$ (where the latter is their attempt at writing 3 as a log term) into one log term. E.g. as shown in the scheme or may see $\log_3(1-x)^2+\log_3"27"\to\log_3("27"(1-x)^2)$ Allow for slips copying terms but must be combining terms of the correct form. $\dfrac{\log_3(32-12x)}{\log_3(1-x)^2}$ is M0

A1: For a correct equation with logarithms removed, any form. Allow with 3^3 in place of 27.

M1: For solving a quadratic equation that has come from a valid attempt to remove logarithms. A valid attempt is one that moves from $\log_3(f(x))=\log_3(g(x))$ to $f(x)=g(x)$ or from $\log_3(f(x))=A$ to $f(x)=3^A$, although allow "recovery" from $\dfrac{\log_3(32-12x)}{\log_3(1-x)^2}=3\to\dfrac{32-12x}{(1-x)^2}=3^3$ for this mark and the next A mark.

If no method of solution of the quadratic is shown then the method is implied by at least one correct solution for their quadratic.

A1: For $x=-\dfrac{1}{9}$ only. The $\dfrac{5}{3}$ must clearly be discarded if seen, e.g by "reject" stated, or crossed out or $x=-\dfrac{1}{9}$ underlined or boxed to in some way indicate it is the only solution.

Question Number	Scheme	Notes	Marks
5(a)	$3(1)^3+A(1)^2+B(1)-10=k$ **or** $3(-1)^3+A(-1)^2+B(-1)-10=-10k$	Attempts $\mathrm{f}(\pm1)=k$ **or** $\mathrm{f}(\pm1)=-10k$	M1
	$A+B-7=k,\ A-B-13=-10k$ $\Rightarrow -10A-10B+70=A-B-13$	Uses $\mathrm{f}(\pm1)=k$ **and** $\mathrm{f}(\mp1)=-10k$ to eliminate *k* and obtains an equation in *A* and *B* only	M1
	$\Rightarrow 11A+9B=83*$	Correct proof with no errors	A1*
			(3)
(b)	$3\left(\frac{2}{3}\right)^3+A\left(\frac{2}{3}\right)^2+B\left(\frac{2}{3}\right)-10=0$	Attempts $\mathrm{f}\left(\frac{2}{3}\right)=0$	M1
	$11A+9B=83$, $12A+18B=246$ $\Rightarrow A=\ldots, B=\ldots$	Solves $11A+9B=83$ simultaneously with *their* equation in *A* and *B*	M1
	A = – 8, *B* = 19	Correct values	A1
			(3)
(c)	$\mathrm{f}(x)=(3x-2)(x^2+\ldots x+\ldots)$	Uses any appropriate method e.g. long division/inspection to obtain x^2+px+q where *p* and *q* are non-zero.	M1
	$g(x)=x^2-2x+5$	Correct expression	A1
			(2)
			Total 8

Notes

(a)

M1: Attempts to apply the remainder theorem for either term – so substitutes ±1 into f and equates to *k* or $-10k$. The powers of ±1 need not be seen. For attempts via long division look for reaching a quotient $3x^2+(A\pm3)x+\ldots$ and remainder $\pm A\pm B-\alpha$ before setting equal to the remainder.

M1: Must have attempted remainder theorem on both terms with opposite sign (ie both f(1) and f(−1) attempted, but may be set to the wrong remainders). Uses their two equations to eliminate the *k* and form an equation in *A* and *B* only. Note: finding a value for *k* (incorrectly) and substituting is M0.

A1*: Correct equation reached from fully correct work.

(b)

M1: Applies the factor theorem with $\mathrm{f}\left(\frac{2}{3}\right)=0$. Must be correct sign here, but no need to evaluate powers for the method mark. Allow if all that is wrong is a slip in one term. The "=0" may be implied by working. For attempts via long division look for quotient $x^2+\frac{1}{3}(A\pm2)x+\ldots$ and then their remainder (in *A* and *B*) set to zero.

M1: Solves their equation and the one from (a) simultaneously. Not dependent so may be scored from an attempt at $\mathrm{f}\left(-\frac{2}{3}\right)=0$. Must have produced a second equation in *A* and *B.* Look for an attempt to match coefficients and add/subtract, or an attempt to substitute for a variable, reaching values for *A* and *B*. If no method is shown, values reached must match their equations.

A1: Correct values found.

(c)

M1: Attempts inspection, factorisation or long division to try and find the factor. Must produce a three-term quadratic. Implied by the correct answer if no incorrect method seen. For inspection look for an attempt to set up and solve at least one equation using coefficients, for factorisation look for correct first and last term, for long division look for correct *x* term for their *A* and a constant (do not be concerned about a remainder for this mark).

A1: Correct g(*x*). May be stated separately or accept if seen in a factorised cubic form $\mathrm{f}(x)=(3x-2)(x^2-2x+5)$ or the correct factor from long division – as long as there is no remainder.

Question Number	Scheme	Notes	Marks
6(a)	Examples: $m_{PQ}=\frac{14+30}{23-15},\ m_{QR}=\frac{-30+26}{15+7}\Rightarrow m_{PQ}\times m_{QR}=..$ or $PQ^2=8^2+44^2, QR^2=22^2+4^2,\ PR^2=30^2+40^2$ $PQ^2+QR^2=..$	Correct strategy to show that $\angle PQR=90°$. E.g. attempts gradient of *PQ* and gradient of *QR* and attempts product or finds side lengths and attempts Pythagoras.	M1
	$m_{PQ}\times m_{QR}=\frac{11}{2}\times\left(-\frac{2}{11}\right)=-1\Rightarrow\angle PQR=90°$ Or $PQ^2=2000,\ QR^2=500,\ PR^2=2500$ $2000+500=2500\Rightarrow\angle PQR=90°$	Correct proof and conclusion	A1
			(2)
(b)(i)	Centre is $(8,-6)$	Correct coordinates	B1
(b)(ii)	$r=\sqrt{(23-8)^2+(14+6)^2}$ or e.g. $r=\frac{1}{2}\sqrt{(23+7)^2+(14+26)^2}$	Fully correct method for the radius	M1
	r = 25	Cao	A1
			(3)
(c)	S is $(1, 18)$ **or** $m_T=\frac{7}{24}$	Correct coordinates for *S* or correct gradient for the tangent.	B1
	$m_N=\frac{18+6}{1-8}\Rightarrow m_T=\frac{8-1}{18+6}\Rightarrow y-18=\frac{7}{24}(x-1)$ Uses a correct straight line method for the tangent using their *S* and the negative reciprocal of the radius gradient		M1
	$7x-24y+425=0$	Allow any integer multiple	A1
			(3)
			Total 8

Notes

(a)

M1: For a correct strategy to show that angle *PQR* = 90°. Possible methods are:
Find gradients of *PQ* and *RQ* (in order to check the perpendicularity condition), getting at least as far as both gradients.
Finding the lengths of all three sides and attempting to show $PQ^2+QR^2=PR^2$ attempting substitution into at least $PQ^2+QR^2=..$ When finding lengths, accept at least two correct lengths stated to imply method for finding lengths if no method is shown.
Finding the lengths of all three sides and applying the cosine rule to find the angle, reaching at least substitution into the cosine rule. May find the two acute angles and subtract from 180°.
Finding the midpoint of *PR* and showing it is equidistant from *P*, *Q* and *R* to deduce *PR* is a diameter and applying circle theorem. Must reach at least the calculations of lengths.
Finding the length of all three sides and calculating angle at *P* from both sin and cosine ratios to show both agree. Must reach angle from both ratios.
Other methods are possible, if you see a method you are unsure of then send to review.

A1: For a fully correct proof with all necessary details shown and some kind of concluding statement made (need not be the final statement). Must include the relevant correct calculations (e.g. showing product equals −1 for gradient approach, evaluation of the squares for Pythagoras etc) and deduce the right angle. If details/calculations are missing, then A0. If rounded values (e.g. when finding values) are used, then A0. Conclusion should refer to the angle, not just *PQ* and *QR* being perpendicular.

(b)

(i)

B1: Correct coordinates for the centre, seen anywhere – may be on the diagram. May have been found in part (a).

(ii)

M1: Correct method for the radius. May have been found in part (a). Implied by correct answer if no method is shown. May be seen on the diagram.

A1: r = 25 (seen anywhere, e.g. on diagram, as long as it is clearly the radius). Isw after a correct radius is found.

(c)

B1: Either identifies the correct point *S*, or finds the correct gradient for the tangent. Look for the gradient they use in their equation.

M1: For a full method to find the equation of the line. E.g. as in scheme, attempts to find the diametrically opposite point to *Q*, finds radius gradient and takes negative reciprocal (oe, may be using centre and either *S* or *Q*) and uses these to form the equation. They must be using an attempt at *S* and not *Q* (but see alt) but accept any attempt that gives a point above the *x*-axis for *S*.

Alternatively, may find the gradient of tangent using centre and *Q* and attempt a translation of the line. Look for

$y=\frac{7}{24}\left(x-15\right)-30 \rightarrow y=\frac{7}{24}\left((x\pm2\times"7")-15\right)-30\pm2\times"24"$ (or may find the vertical translation first via trig or Pythagoras).

For attempts at the gradient look for an attempt at the radius followed by negative reciprocal, or an attempt at change in *x* over change in *y* directly. Allow if there are sign slips if the intent is clear. Some may attempt differentiation, look for evidence of implicit differentiation giving a $y\frac{\mathrm{d}y}{\mathrm{d}x}$ term before attempting to substitute *x* and *y* values.

A1: $7x-24y+425=0$ or any (non-zero) integer multiple of this. Accept terms in any order, but have the “=0”.

Question Number	Scheme	Notes	Marks
7(i)	$3\sin(2x-15°)=\cos(2x-15°)$ $\Rightarrow \tan(2x-15°)=\frac{1}{3}$	Uses $\tan\theta\equiv\frac{\sin\theta}{\cos\theta}$ and reaches $\tan(2x-15°)=\ldots$	M1
	$\Rightarrow x=\frac{\tan^{-1}\left(\frac{1}{3}\right)\pm15°}{2}$	Correct strategy for finding *x*	M1
	$x=16.7°,\ -73.3°$	One of awrt 16.7 **or** –73.3	A1
		Awrt 16.7 **and** awrt –73.3 and no extras in range	A1
			(4)
(ii)	$4\sin^2\theta+8\cos\theta=3$ $\Rightarrow 4(1-\cos^2\theta)+8\cos\theta=3$ $\Rightarrow 4\cos^2\theta-8\cos\theta-1=0$	Applies $\sin^2\theta=1-\cos^2\theta$ and collects terms to obtain a 3TQ in cos *θ*	M1
	$\cos\theta=\frac{8\pm\sqrt{64+4\times4}}{2\times4}\Rightarrow\theta=\cos^{-1}\left(1-\frac{\sqrt{5}}{2}\right)=\ldots$	Solves their 3TQ and takes inverse cos to obtain at least one value for *θ*	M1
	$\theta=1.69,\ 4.59$	Awrt 1.69 **or** 4.59	A1
		Awrt 1.69 **and** awrt 4.59 and no extras in range	A1
			(4)
			Total 8

Notes (i)

M1: Evidence of correct identity $\tan\theta=\frac{\sin\theta}{\cos\theta}$ seen, with attempt made to reach an equation $\tan(2x-15°)=k$. Note $\tan(2x-15°)=3$ with no correct identity stated is M0, but if correct identity stated first allow M1 as a slip. Allow recovery for all relevant marks for $3\tan(2x-15°)=0\rightarrow\tan(2x-15°)=\frac{1}{3}$ but allow only the first M if $\tan(2x-15°)=0$ follows.
Allow with e.g. $\tan y=\ldots$ as long as *y* has been clearly defined as 2*x* − 15°.
Alternatively, may square both sides (including the 3) and apply $\sin^2\theta+\cos^2\theta=1$ to reach an equation in sin or cos. They should get $\sin(2x-15°)=\pm\frac{1}{\sqrt{10}}$ or $\cos(2x-15°)=\pm\frac{3}{\sqrt{10}}$ if correct but allow the method if slips in rearranging are made.

M1: Correct order of operations from $\tan(2x-15°)=k$ (any non-zero *k*) to produce a value of *x*, so applies arctan, then attempts to move the 15° across before dividing by 2. Must be working in degrees OR entirely in radians (if the 15° is converted first).

A1: One of awrt 16.7 **or** –73.3. Must have come from a correct equation – answer from calculators score A0 (they will not have score the M).

A1: Both of awrt 16.7 **and** –73.3 and no other solutions in the range. Must have come from a correct equation – answer from calculators score A0 (they will not have scored the M).

(ii)

M1: Applies $\sin^2\theta=1-\cos^2\theta$ and collects terms to obtain a 3TQ in cos *θ* or rearranges suitably to be able to solve via completing the square (oe method). Allow the method marks if there are notation errors (e.g. missing *θ*.

M1: Solves their 3TQ and takes inverse cos to obtain at least one value for θ (may be implied by answers to 1d.p.). Usual rules for solving. Answers in degrees can score this method mark for find one angle.

A1: One correct answer, awrt 1.69 or awrt 4.59. Must be in radians.

A1: Both correct, awrt 1.69 and awrt 4.59, and no other solutions in the range.

Question Number	Scheme	Notes	Marks
8(a)	$u_{20} = 100 + 19(-2) = 62\,*$	Correct method shown	B1*
			(1)
(b)	$S_{20} = \frac{1}{2}(20)\{2\times100+19(-2)\} = \ldots$ **or** $S_{20} = \frac{1}{2}(20)\{100+62\} = \ldots$	Applies a correct AP sum formula with n = 20, a = 100 and d = –2 **or** n = 20, a =100 and l = 62	M1
	$= 1620$ (mm)	Correct value	A1
			(2)
(c)	$62 \times r^2 = 60 \Rightarrow r^2 = \ldots$	Correct strategy to find r	M1
	$r^2 = \frac{60}{62} \Rightarrow r = \sqrt{\frac{60}{62}}$ = 0.983738...	r = awrt 0.984	A1
			(2)
(d)	Total distance from GS hits = $\frac{62\times0.983\ldots(1-0.983\ldots^n)}{1-0.983\ldots}$		M1
	$1620 + \frac{62\times0.983\ldots(1-0.983\ldots^n)}{1-0.983\ldots} > 3000$	Correct equation set up with their r and suitable a	M1
	$0.983\ldots^n < 0.63207\ldots \Rightarrow n = \frac{\log(0.63207\ldots)}{\log(0.983\ldots)}$	Fully correct processing to find n from an equation of suitable form,	M1
	$n = 27.98\ldots \Rightarrow N = 20 + 28 = 48$	N = 48 only	A1cso
			(4)
	(d) Alternative taking 20th hit as first term of GP		
	Total distance from GS hits = $\frac{62(1-0.983\ldots^n)}{1-0.983\ldots}$		M1
	$1620 - 62 + \frac{62(1-0.983\ldots^n)}{1-0.983\ldots} > 3000$	Correct equation set up with their r and suitable a	M1
	$0.983\ldots^n < 0.62179\ldots \Rightarrow n = \frac{\log(0.62179\ldots)}{\log(0.983\ldots)}$	Fully correct processing to find n from an equation of suitable form,	M1
	$n = 28.98\ldots \Rightarrow N = 19 + 29 = 48$	N = 48 only	A1cso
			Total 9

Notes

(a)

B1*: Correct method shown, identifies the common difference and attempts the 20th term.
Accept as minimum seeing $100 + 19(-2) = 62$.
Alternatively, allow for setting up an equation $62 = 100 + (n-1)(-2)$ and solving to find n = 20
If listing, there must be the correct 20 terms with final term 62.

(b)

M1: Applies a correct AP sum formula with n = 20, a = 100 and d = –2 or with n = 20, a =100 and l = 62. Alternatively, by listing look for 20 terms listed with attempt to sum, or implied by answer.

A1: Correct value, units not needed.

(c)

M1: Correct strategy, attempts to set up and solve the equation $62\times r^2 = 60$

A1: *r* = awrt 0.984. Accept the exact answer $\sqrt{\dfrac{30}{31}}$ (simplified to this)

(d)

This is now being marked as MMMA (on ePen it isMMAA).

M1: Applies a correct GS summation with their *r*, *a* = 62 or $62\times \text{their } r$ and *n* or *n* − 1 or *n* – 19 or *n* – 20 as part of their solution in (d). Special case, allow with *a* =100 for this mark if they have used this in part (c) as a misunderstanding of the question.

M1: States or uses $1620+\dfrac{62\times 0.983...\left(1-0.983...^n\right)}{1-0.983...}>3000$ with their 1620 from part (b) and their *r* with *n* or *n* – 1 or *n* – 19 or *n* – 20. Allow " > ", " = ", " < " etc. but the starting term of the GS for the combination of series must be correct.

Alternatively States or uses $1620-62+\dfrac{62\left(1-0.983...^n\right)}{1-0.983...}>3000$ with their 1620 from part (b) and their *r* with *n* or *n* – 1 or *n* – 19 or *n* – 20. Allow " > ", " = ", " < " etc. but the starting term of the GS for the combination of series must be correct.

M1: Correct processing to solve for *n* in their equation which must be an attempt at combining 3000, their (b) and the sum of a GS with their *r* and *a* = 62 or $62\times \text{their } r$ and *n* or *n* − 1 or *n* – 19 or *n* – 20. Look for reaching $(\text{their } r)^n = ...$ before applying logs appropriately to proceed to *n* =... (Note *n* = 27.98 or 28.98 are possible correct values at this stage.)
Allow for any of " > ", " = ", " < " used in the equations.

A1cso: For *N* = 48 only and must have come from a correct equation (although allow for any of " > ", " = ", " < ") – all three M's must have been earned. Look for the correct values of *n* as a clue. If using power and *n* – 19 or *n* – 20 they may get directly to the answer.

Note: It is possible to get the correct answer from incorrect methods and you may see M1M0M1A0 often. Make sure they are working from a correct equation. The values 27.98 and 28.98 may help (if full accuracy is kept). A value between 27.2 and 27.8 (depending on degree of rounding used) before adding 20 likely implies a mismatch between starting term of GS and ending of AS so check carefully if the second M was earned.

Question Number	Scheme	Notes	Marks
9(a)	$mx = x - x^2 \Rightarrow m = 1 - x \Rightarrow x = \ldots$ Or $y = \frac{y}{m} - \frac{y^2}{m^2} \Rightarrow m^2 = m - y \Rightarrow y = \ldots$	Attempts to eliminate either *x* or *y* and factors out or cancels *x*/*y* to get a linear equation and solve.	M1
	$x = 1 - m$ **and** $y = m(1-m)$	Both correct	A1
			(2)
(b)	$\int x - x^2 (-mx)\,\mathrm{d}x = \frac{x^2}{2} - \frac{x^3}{3}\left(-m\frac{x^2}{2}\right)$	$x^n \to x^{n+1}$ in at least one term	M1
	Area of $R_1 = \int_0^{"1-m"} \{x - x^2(-mx)\}\,\mathrm{d}x$ $= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3}\left(-m\frac{(1-m)^2}{2}\right) - 0$	Uses the limits "1 – *m*" and 0 in their integrated expression and subtracts (condone the omission of the "– 0")	M1
		Correct expression in *m* with/without the area under line subtracted.	A1
	Area of $R_1 = \int_0^{"1-m"} \{x - x^2 - mx\}\,\mathrm{d}x = \frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3}(-0)$ Correct strategy for the area (may be scored for finding separate areas and subtracting)		dM1
	$= \frac{(1-m)^3}{6}$ *	Correct expression	A1*
			(5)
(c)	Area of $(R_1 + R_2) = \int_0^1 (x - x^2)\,\mathrm{d}x = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \ldots$ $\left(= \frac{1}{6}\right)$	Correct method for finding the area of $R_1 + R_2$ Alternatively, a correct method for finding the area of R_2	M1
	Alt: Area of $R_2 = \int_{1-m}^1 (x - x^2)\,\mathrm{d}x + \frac{1}{2}("1-m") \times m(1-m)$ $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1-m}^1 + \frac{1}{2}m(1-m)^2 = \ldots \left(= \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2}m(1-m)^2\right)$		
	$R_1 = R_2 \Rightarrow \frac{(1-m)^3}{6} = \frac{1}{12} \Rightarrow m = \ldots$	Sets up a correct equation using the answer to part (b) and solves for *m*	dM1
	Alt: $\frac{(1-m)^3}{6} = \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2}m(1-m)^2 \Rightarrow m = \ldots$		
	$m = 1 - \frac{1}{\sqrt[3]{2}}$	Correct exact value in any form	A1
			(3)
			Total 10

Notes

(a)

M1: Attempts to eliminate either *x* or *y* and factors our or cancels *x*/*y* to get a linear equation and solves the resulting equation. May be implied by one correct coordinate of the two.

A1: Both coordinates correct. Accept $y = m - m^2$

(b)

M1: Attempts to integrate equation for curve *C* to find the area, may be part of integrating a difference of curve and line, or may be done separately.

M1: Applies the limits of 0 (may be implied) and their *x* from (a) (which must be in terms of *m*) to the integral (which must a changed function), and subtracts the correct way round. Again, may or may not be including the line at this point.

A1: Correct expression in *m* for the area under *C* between 0 and 1 – *m*. If the line equation has been subtracted and combined already, then it is for an unsimplified correct overall expression, but if it is separate then the A1 can be scored just the for area under *C*. If combined it is $\text{Area} = \frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3}(-0)$

dM1: Depends on previous M. Correct overall strategy for the area. Can be scored if they have attempted curve – line, or may be scored for finding separate areas and subtracting the triangle area from area under curve. For the area under line, if not as part of integral, look for $\frac{1}{2} \times \text{their } x \times \text{their } y$ from part (a) subtracted from the curve.

A1: Fully correct work leading to the given answer with sufficient evidence of combination of terms before the given answer. Look for reaching an expression of the form $\frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3}$ with $(1-m)^3$ terms before the final answer. Going from $= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3} - m\frac{(1-m)^2}{2}$ to the given answer with no intermediate step is A0.
Alternatively, they may fully expand their expression and factorise, or fully expand their expression and the given answer and compare and conclude it is the same – check carefully that all working is correct.

(c)

M1: Correct method for finding the area of $R_1 + R_2$. Look for the integral of *C* with limits 0 to 1 applied, the subtraction of 0 may be implied. Allow for $\frac{1}{6}$ from a correctly set up integral if the integration is not shown.
Alternatively score for a correct method for finding the area of R_2. Look for the integral of *C* from 1 – *m* to 1 with the area of the triangle added.

dM1: Sets up a correct equation using the answer to part (b) and reaches a value for *m*.

A1: $m = 1 - \frac{1}{\sqrt[3]{2}}$ Correct exact value in any form

Question Number	Scheme	Notes	Marks
10(i)	E.g. $p=7 \Rightarrow 2p+1=15$ Which is not a prime number (so the statement is not true)	Identifies a counter example and makes a conclusion/shows it is not prime.	B1
			(1)
(ii)	n odd $\Rightarrow n=2k+1$ $\Rightarrow 5n^2+n+12=5(2k+1)^2+2k+1+12$ **or** n even $\Rightarrow n=2k$ $\Rightarrow 5n^2+n+12=5(2k)^2+2k+12$	Starts the proof by considering *n* odd **or** *n* even and substituting into the expression (see notes for logical approach)	M1
	n odd $\Rightarrow n=2k+1$ $\Rightarrow 5n^2+n+12=5(2k+1)^2+2k+1+12$ **and** n even $\Rightarrow n=2k$ $\Rightarrow 5n^2+n+12=5(2k)^2+2k+12$	Considers *n* odd **and** *n* even (as above)	M1
	n odd : $20k^2+22k+18$ which is even and n even : $20k^2+2k+12$ which is even	Attempts both with **at least one correct** expression that is stated to be even	A1
	n odd : $2(10k^2+11k+9)$ and n even : $2(10k^2+k+6)$ These are both even so $5n^2+n+12$ must be even for all integers *n*	Fully correct proof that considers both *n* odd and *n* even, shows the resulting expressions are even and makes a suitable conclusion	A1
			(4)
			Total 5

Notes

(i)

B1: For any correct counter example shown with calculation and suitable conclusion.
Another common suitable example is *p* = 13, as $2\times13+1=27=3^3$ so not prime.
Accept "not prime" or showing not prime by indicating a factor as conclusion.
E.g. a minimal acceptable answer is "$2\times7+1=15$ not prime", or "$2\times7+1=15=5\times3$"

(ii) **Note** No marks if all they do is try a few cases.

M1: Considers the case of *n* being odd or *n* being even by setting it up algebraic substituting *n* = 2*k* or $n=2k\pm1$ into the quadratic expression. Allow with variables other than *k* – including *n*.

M1: Considers both odd and even cases, same criteria for each as in the first M.

A1: Both cases attempted with at least one correct expression (allow with *n*) simplified to a point where all coefficients are even integers with deduction made that the expression is even.
Note for $n=2k-1$ it is $20k^2-18k+16=2(10k^2-9k+8)$

A1: Complete proof with all work correct and with factor 2 seen extracted in both expressions or explanation that all terms are multiples of 2/even hence it is even, and conclusion that the given expression is even for all integers. **Must be using a variable other than *n* for this mark.**

For approaches via "logic" a maximum of M1M1A0A0 is possible unless all relevant "odd × odd = odd" properties are also proved (which is unlikely to occur).

M1: Attempts logic reasoning that builds by term and gets at least as far as explaining a product of two terms, so "if *n* is odd/even then n^2 is odd/even" or equivalent if they have

$n(5n+1)+12$ would require reasoning such as if *n* is odd 5n + 1 is even so *n*(5n + 1) is even.)

M1: Attempts logic reasoning for both odd and even cases, same criteria as for the first M.

A1: Attempts both cases with a full reasoning of each terms that includes proofs of all relevant properties that odd × odd = odd, even + even = even and so on.

A1: Full reasoning for both cases given with conclusion drawn.