



Mark Scheme (Results)

Summer 2019

Pearson Edexcel IAL Mathematics (WMA11/01)

Pure Mathematics

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Publications Code WMA11_01_1906_MS

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Pearson Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\sqrt{}$ or ft will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.
8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1.(a)	$\frac{dy}{dx} = \frac{1}{8} \times 3x^2 - 24x - \frac{1}{2}x^{-\frac{3}{2}}$ $\frac{dy}{dx} = \frac{3}{8}x^2 + 12x^{-\frac{3}{2}}$	M1 A1 A1 (3)
(b)	$\left. \frac{dy}{dx} \right _{x=4} = \frac{3}{8} \times 4^2 + 12 \times 4^{-\frac{3}{2}} = (7.5)$ $y + 3 = 7.5(x - 4) \Rightarrow y = 7.5x - 33$	M1 M1 A1 (3) (6 marks)

(a)

M1 For reducing a correct power by one on either x term. Allow for either $x^3 \rightarrow x^{3-1}$ or $x^{-\frac{1}{2}} \rightarrow x^{-\frac{1}{2}-1}$
 So this **cannot** be awarded for $\frac{24}{x^{0.5}} \rightarrow \frac{24}{0.5x^{-0.5}}$

A1 One term of $\frac{3}{8}x^2 + 12x^{-\frac{3}{2}}$ correct and simplified

A1 $\frac{dy}{dx} = \frac{3}{8}x^2 + 12x^{-\frac{3}{2}}$ or exact simplified equivalent such as $\frac{dy}{dx} = \frac{3}{8}x^2 + \frac{12}{x\sqrt{x}}$.
 Isw after a correct answer. There is no need to see the $\frac{dy}{dx}$

(b)

M1 For attempting to find the value of their $\frac{dy}{dx}$ at $x = 4$.

Score for sight of embedded 4's followed by an answer. Condone slips

If no calculations are seen then only award if the value is correct for their $\frac{dy}{dx}$

M1 For correct method of finding the equation of the tangent at $(4, -3)$ using their numerical $\left. \frac{dy}{dx} \right|_{x=4}$

Condone one error on the sign of the 4 and -3 within the tangent formula.

If the form $y = mx + c$ is used they must proceed as far as $c = \dots$

It cannot be awarded from a made-up gradient

A1 $y = 7.5x - 33$ or exact equivalent in the form $y = mx + c$. ISW after a correct answer.

If the form $y = mx + c$ is used they must get correct m and c and write

$y = 7.5x - 33$ oe

NB. If a calculator is used to find $\left. \frac{dy}{dx} \right|_{x=4} = 7.5$ **without sight of** $\frac{dy}{dx} = \frac{3}{8}x^2 + 12x^{-\frac{3}{2}}$ then you may allow the final two marks in (b) for correct method to find a correct tangent.

Question Number	Scheme	Marks
2.(a)	$\frac{1}{4-2\sqrt{2}} = \frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$ $= \frac{4+2\sqrt{2}}{16-8} = \frac{1}{2} + \frac{1}{4}\sqrt{2} \quad \text{oe}$	M1 A1 (2)
(b)	$4x = 2\sqrt{2}x + 20\sqrt{2} \Rightarrow (4-2\sqrt{2})x = 20\sqrt{2}$ $\Rightarrow x = \frac{20\sqrt{2}}{(4-2\sqrt{2})} = 20\sqrt{2} \times (a)$ $\Rightarrow x = 20\sqrt{2} \times \left(\frac{1}{2} + \frac{1}{4}\sqrt{2}\right) = 10 + 10\sqrt{2}$	M1 dM1 A1 (3) (5 marks)

(a)

M1 For sight of $\frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$ oe

A1 For achieving $\frac{1}{2} + \frac{1}{4}\sqrt{2}$ or exact equivalent such as $0.5 + \frac{\sqrt{2}}{4}$, $\frac{2}{4} + \frac{2}{8}\sqrt{2}$ or correct a and b .

Remember it does not have to be simplified and isw following a correct answer

(b) Hence

M1 For attempting to collect the terms in x on one side of the equation and the constant term on the other side. Condone slips but there must be an attempt to collect terms with a bracket or implied bracket

dM1 For using part (a) and attempting to find $k\sqrt{2} \times (a)$

A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ but NOT $10(\sqrt{2} + 1)$. It cannot be awarded without sight of $k\sqrt{2} \times (a)$

Otherwise (1)- SQUARING APPROACH

M1 Squaring both sides $4x = 2\sqrt{2}x + 20\sqrt{2} \rightarrow 16x^2 = 8x^2 + 160x + 800$ Condone slips on coefficients
 Cannot be scored by squaring each term. Look $ax^2 = px^2 + qx + r$

dM1 Re-arranging and attempting to solve their 3TQ usual rules

$$\text{Eg } 8x^2 - 160x - 800 = 0 \Rightarrow x^2 - 20x - 100 = 0 \Rightarrow x = \frac{20 \pm \sqrt{400 + 400}}{2}$$

A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ following a correct solution of the quadratic equation seen above.

Otherwise (2)- REPEATING THE PROCESS OF PART (a)

M1 Rearranges $4x = 2\sqrt{2}x + 20\sqrt{2} \Rightarrow (4 \pm 2\sqrt{2})x = 20\sqrt{2}$ condoning slips. May even divide by 2 first

dM1 Then divide, rationalise and attempt to simplify. Eg $x = \frac{20\sqrt{2}}{(4-2\sqrt{2})} \times \frac{(4+2\sqrt{2})}{(4+2\sqrt{2})} = \frac{80\sqrt{2} + 40\sqrt{2} \times \sqrt{2}}{16-8}$ oe

A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ only. It cannot be awarded without sight of the correct intermediate line seen above

Question Number	Scheme	Marks
3.(a)	Attempts perimeter of garden = $2 \times 5x + 2 \times (6x - 2)$ Sets $2 \times 5x + 2 \times (6x - 2) > 29 \Rightarrow 22x > 33$ $\Rightarrow x > \frac{33}{22} \Rightarrow x > 1.5$ *	M1 dM1 A1* (3)
(b)	Attempts area of garden = $2x(2x - 1) + 3x(6x - 2)$ Sets $A < 72 \Rightarrow 22x^2 - 8x - 72 < 0$ Finds critical values $11x^2 - 4x - 36 \Rightarrow x = -\frac{18}{11}, 2$ Chooses inside region $-\frac{18}{11} < x < 2$	M1 A1 M1 ddM1 A1 (5)
(c)	$1.5 < x < 2$	B1 (1)
		(9 marks)

(a)

- M1 An attempt at finding the perimeter of the garden.
Scored for sight of $5x + 2x - 1 + 2x + 6x - 2 +$ additional term(s) involving x
Individual lengths may not be seen so imply for sight of a total of $ax + b$, where $a > 15$
- dM1 Sets their $P > 29$ and attempts to solve by proceeding to $ax > c$
You may condone an attempt in which $P = 29 \Rightarrow ax = c$

- A1* cso with at least one correct intermediate (simplified) line $22x > 33$ or $x > \frac{33}{22}$ before $x > 1.5$ seen.
Condone an attempt in which you see $P = 29 \Rightarrow x = 1.5$ before $x > 1.5$ seen
Note that it is possible to start with $x > 1.5$ and prove $P > 29$ but for the A1* to be scored there must be a final statement of the type "hence $x > 1.5$ ". There is no requirement for any units

(b) **Mark part (b) and (c) together**

- M1 For an attempt at finding the area of the garden. For this to be scored look for
The sum of two areas $2x(2x - 1) + \dots x(6x - 2)$ condoning slips
The sum of two areas $5x(2x - 1) + \dots x(\dots x \pm \dots)$ condoning slips
The difference between two areas $5x(6x - 2) - 2x(\dots x \pm \dots)$ condoning slips.

- A1 A "correct and simplified" equality or inequality, condoning $< \leftrightarrow \leq \leftrightarrow =$ Eg. $22x^2 - 8x - 72 < 0$ or

- M1 A valid attempt to find the critical values of their 3TQ. Allow factorisation, formula, completion of square or use of calculator. If a calculator is used then the answer(s) must be correct for their 3TQ.
Condone candidates who fail to state the negative root of their quadratic.

- ddM1 Dependent upon both M's. For choosing the inside region for their critical values. Condone $< \leftrightarrow \leq$
Condone for this mark replacing a negative root with 0, 0.5 or 1.5. So accept for example one of either $1.5 < x < 2$, $0 < x < 2$ or $0.5 < x < 2$

- A1 $-\frac{18}{11} < x < 2$ Allow $0 < x < 2$ or $0.5 < x < 2$ due to context. Allow alternative notation. See below

(c)

B1 $1.5 < x < 2$. Accept versions such as $(1.5, 2)$, $x > 1.5$ **and** $x < 2$, $x > 1.5 \cap x < 2$

Do not allow $x > 1.5$ or $x < 2$ $x > 1.5, x < 2$

Question Number	Scheme	Marks
4.	$\frac{4x^2+1}{2\sqrt{x}} = \frac{4x^2}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} = 2x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$ $\int \frac{4x^2+1}{2\sqrt{x}} dx = \frac{4}{5}x^{\frac{5}{2}} + x^{\frac{1}{2}} + c$	M1 A1 M1 A1 A1 (5 marks)

M1 Attempts to write $\frac{4x^2+1}{2\sqrt{x}}$ as a **sum of two terms**.

Award if any index is correct and processed for a form $Px^m + Qx^n$

Do not allow if the indices are unprocessed Eg. $\frac{x^2}{\sqrt{x}} = x^{2-\frac{1}{2}}$

A1 $2x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$ oe. The coefficients do not have to be simplified. Condone $\frac{4x^{\frac{3}{2}} + x^{-\frac{1}{2}}}{2}$ for M1 A1

M1 Raises the power of one **correct** index by one.

The index must be processed correctly so award for sight of $\rightarrow \dots x^{\frac{5}{2}}$ or $\dots x^{\frac{1}{2}}$

A1 For one correct term and in simplest form. Either $+\frac{4}{5}x^{\frac{5}{2}}$ or $+x^{\frac{1}{2}}$

A1 Fully correct $\frac{4}{5}x^{\frac{5}{2}} + x^{\frac{1}{2}} + c$. Accept exact equivalent simplified answers such as $0.8x^2\sqrt{x} + \sqrt{x} + c$

Condone spurious notation such as $\int$ or a dx

Condone $\frac{4}{5}x^{\frac{5}{2}} + 1x^{\frac{1}{2}} + c$

Question Number	Scheme	Marks
5.(a)	$2x^3 + 3x^2 - 35x = 0 \Rightarrow x(2x^2 + 3x - 35) = 0$ $(2x - 7)(x + 5) = 0 \Rightarrow x = \dots$ $x = -5, 0, \frac{7}{2}$	M1 dM1 A1 (3)
(b)	$2(y - 5)^6 + 3(y - 5)^4 - 35(y - 5)^2 = 0$ States that $y = 5$ is a solution $(y - 5)^2 = \frac{7}{2} \Rightarrow y = \dots$ $y = 5 + \sqrt{\frac{7}{2}}$ or $y = 5 - \sqrt{\frac{7}{2}}$ or exact equivalent Both $y = 5 + \sqrt{\frac{7}{2}}$ and $y = 5 - \sqrt{\frac{7}{2}}$ or exact equivalent.	B1 M1 A1ft A1 (4) (7 marks)

(a)

M1 Takes out a common factor of x. Score if each term is divided by x.

dM1 Attempts to solve the resulting quadratic **via algebra** (usual rules). Allow factorisation, formula or completion of square. They cannot just write down answers from their calculator for this mark.A1 $x = -5, 0, \frac{7}{2}$

Note 1: Some candidates will just write down their answers from a calculator. This scores 0,0,0

Note 2: Some students will attempt to solve the cubic by the quadratic formula Eg.

$$2x^3 + 3x^2 - 35x = 0 \Rightarrow a = 2, b = 3, c = -35 \text{ and use } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = -5, \frac{7}{2}$$

This scores 0,0,0 as the method used is incorrect

(b)

B1 States that 5 is a solution of the given equation in (b)

M1 Realises that $x = (y - 5)^2$ and proceeds to find a value for y using $(y - 5)^2 = \frac{7}{2} \Rightarrow y = \dots$ Follow through on any positive value from (a). Allow decimal answers here. Don't be overly concerned by the mechanics of their solution.

A1ft A solution of $5 + \sqrt{\frac{7}{2}}$ or $5 - \sqrt{\frac{7}{2}}$ You should follow through on their positive root.
Allow decimals for this mark only. So accept awrt 6.87 or awrt 3.13

A1 Both $5 + \sqrt{\frac{7}{2}}$ and $5 - \sqrt{\frac{7}{2}}$ with no other solutions for part (b) apart from 5. Do not allow decimal equivalents. Don't allow complex solutions.

Question Number	Scheme	Marks
6.(a)	Sets $4x + c = x(x - 3)$ and attempts to write as a 3TQ Uses $b^2 = 4ac$ for their $x^2 - 7x - c = 0$ Correct equation $49 = -4c$ or $49 + 4c = 0$ $c = -12.25$ oe	M1 dM1 A1 A1 (4)
(b)	Attempt to solve $x^2 - 7x - c = 0$ with their c Attempt to find the y coordinate for their x coordinate $\left(\frac{7}{2}, \frac{7}{4}\right)$ oe	M1 dM1 A1 (3) (7 marks)

- (a)
- M1 Sets $4x + c = x(x - 3)$ and attempts to write as a 3TQ. All terms don't need to be on the same side of the equation and you should condone slips
- dM1 Attempts to use $b^2 = 4ac$ or $b^2 - 4ac = 0$ for their 3TQ. This may be implied by later work.
For a correct 3TQ, errors on "c" will be common, so condone $\pm 49 \pm 4c = 0$ oe
- A1 Correct equation formed in 'c'
- A1 Correct solution $c = -12.25$ oe
- (b)
- M1 Attempts to solve their $x^2 - 7x - c = 0$ with their c ($c \neq 0$)
Allow usual methods. If a calculator is used then the answer(s) must be correct for their 3TQ
Incorrect values of c may produce two values of x
- dM1 Attempt to find the y coordinate from their x coordinate. It is dependent upon the previous M
If there are two coordinates they only need to find the y value for one of their x values to score this mark.
- A1 Correct coordinate given $\left(\frac{7}{2}, \frac{7}{4}\right)$ or exact equivalent. Allow written separately $x = 3.5, y = 1.75$
If c is correct allow the answer to score all 3 marks as long as no incorrect working is seen.

.....
Alternative to (a) by using gradients

- M1 Attempts to set $\frac{d}{dx}(x^2 - 3x) = 4 \Rightarrow 2x - 3 = 4 \Rightarrow x = \dots$
- dM1 Attempts to find the y coordinate by substituting this answer in $y = x(x - 3)$
- A1 Correct equation in c Eg $\frac{7}{4} = 4 \times \frac{7}{2} + c$
- A1 Correct solution $c = -12.25$ oe

Note: Using this method the answer to (b) is found before (a). Some candidates will not realise this however. For the marks in (b) to be awarded there must be a statement somewhere in the solution to the effect that they know that they have solved part (b). This could be simply

- (b) (3.5, 1.75) or coordinates of point of intersection is $\left(\frac{7}{2}, \frac{7}{4}\right)$

Question Number	Scheme	Marks
7.(a)	Attempts to use $\frac{1}{2}r^2\theta$ with $r = 6$ and any allowable angle θ Full method to find area $\frac{1}{2} \times 6^2 \times (2\pi - 0.7)$ or $\pi \times 6^2 - \frac{1}{2} \times 6^2 \times 0.7$ $= 100.5 \text{ cm}^2$ (awrt)	M1 M1 A1 (3)
(b)	Attempts $\frac{\sin \angle ADO}{6} = \frac{\sin 0.7}{5} \Rightarrow \sin \angle ADO = 0.77...$ $\angle ADO = 2.258$ (awrt)	M1 A1 A1 (3)
(c)	Attempts arc length $ABC = 6 \times (2\pi - 0.7)$ 33.50 Attempts length OD $\frac{\sin(\pi - 0.7 - "2.258")}{OD} = \frac{\sin 0.7}{5} \Rightarrow OD = ...$ 1.42 Full method to find perimeter = "33.50" + 5 + 6 - "1.42" $= 43.1 \text{ cm}$	M1 M1 ddM1 A1 (4)
Alt (c)	Alternative for arc length $ABC = 12\pi - 6 \times 0.7$ Alternative for finding OD using the cosine rule $OD^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \cos(\pi - 0.7 - "2.258") \Rightarrow OD$	M1 M1
7.(a)	Solutions where candidate changes to degrees Look for angle $AOD =$ awrt 40° to score M marks Attempts to use $\frac{\theta}{360} \pi r^2$ with $r = 6$ and angle $\theta =$ awrt 40 or 320 Full method to find area $\frac{(360 - \text{awrt } 40)}{360} \times \pi 6^2$ or $\pi \times 6^2 - \frac{\text{awrt } 40}{360} \times \pi 6^2$ $= 100.5 \text{ cm}^2$ (awrt)	M1 M1 A1 (3)
(b)	Attempts $\frac{\sin \angle ADO}{6} = \frac{\sin 40^\circ}{5} \Rightarrow \sin \angle ADO = 0.77...$ $\angle ADO = 129.4^\circ$ (awrt)	M1 A1 A1 (3)
(c)	Attempts arc length $ABC = \frac{(360 - 40)}{360} \times 2\pi 6$ 33.50 Attempts length OD $\frac{\sin(180 - 40 - "129.4")}{OD} = \frac{\sin 40}{5} \Rightarrow OD = ...$ 1.42 Full method to find perimeter = "33.50" + 5 + 6 - "1.42" $= 43.1 \text{ cm}$	M1 M1 ddM1 A1 (4)
		(10 marks)

Notes

(a)

M1 Attempts to use $\frac{1}{2}r^2\theta$ with $r = 6$ and any angle allowable angle θ

Allowable angles are; 0.7 $\pi - 0.7 = \text{allow awrt } 2.4$ $2\pi - 0.7 = \text{allow awrt } 5.6$

M1 A correct attempt to find the area of sector $ABCOA$. See scheme Accept awrt 100 or 101 for this mark
A1 awrt $100.5(\text{cm}^2)$ The units are not required

(b)

M1 Attempts the sine rule with the lengths and angles in the correct positions $\frac{\sin \angle ADO}{6} = \frac{\sin 0.7}{5}$

A1 Correct value for $\sin \angle ADO = 0.77\dots$. Be careful here! $\angle ADO = 0.77\dots$ is A0
May be implied by either a correct answer or awrt 0.88

A1 awrt $\angle ADO = 2.258$ or 129.4°

(c)

M1 A correct method to find arc length ABC May be implied by sight of $6 \times \text{awrt } 5.6$ or awrt 33.5 or 33.6

M1 A correct method to find length OD using either the sine rule or cosine rule. The angle OAD must be attempted using a correct method ($\pi - 0.7 - "2.258"$).

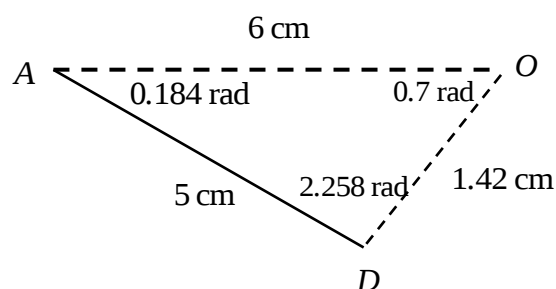
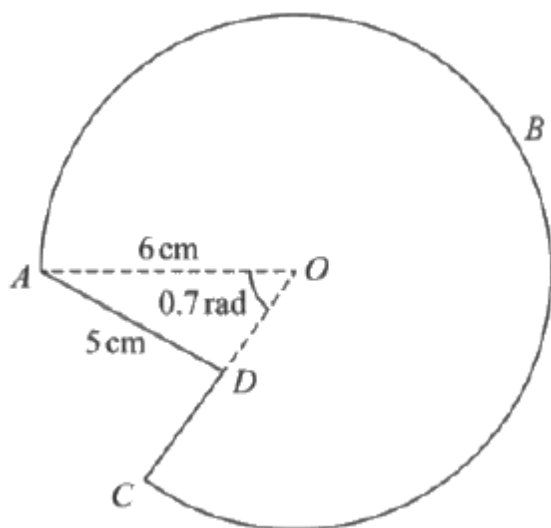
Eg. For the sine rule $\frac{OD}{\sin(\pi - 0.7 - "2.258")} = \frac{5}{\sin 0.7} = \frac{6}{\sin "2.258"} \Rightarrow OD = \dots$

For cosine rule it could be $5^2 = 6^2 + x^2 - 2 \times 6 \times x \times \cos(0.7) \Rightarrow 3\text{TQ in } x$ which must be solved by correct methods

ddM1 Both previous M's must have been scored. It is for a correct method to find the perimeter of the shape.
Look for " 33.5 " + $5 + 6$ - " 1.42 ". It is implied by awrt 43

A1 cso and cao 43.1 cm. Units are not required

Handy Diagrams



Question Number	Scheme	Marks
8.(a)	Substitutes $x=4$ in $f'(4) = 4 \times 2 - 2 - \frac{8}{3 \times 4^2} = \left(\frac{35}{6}\right)$ Attempts to find the gradient of the perpendicular $= -\frac{6}{35}$ Attempts the normal $y-1 = -\frac{6}{35} \times (x-4) \Rightarrow 6x + 35y - 59 = 0$	M1 dM1 M1A1 (4)
(b)	$f'(x) = 4x^{\frac{1}{2}} - 2 - \frac{8}{3x^2} \Rightarrow f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x} (+c)$ $x=4, f(x)=1 \Rightarrow 1 = \frac{8}{3} \times 8 - 8 + \frac{2}{3} + c \Rightarrow c = \dots (-13)$ $f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x} - 13$	M1 A1 A1 dM1 A1 (5) (9 marks)

Notes

Points for marking:

- If all a candidate does is integrate, then mark as though they are attempting (b) even though their solution may have part (a) marked in the margin
- If a candidate differentiates $f'(x)$ in (a) and integrates $f''(x)$ in (b) getting back to the same or similar expression it is 0 marks in (b)
- See Practice items. Read annotations carefully to understand how to apply these principles.
- If in any doubt ask your TL for advice

(a)

M1 Attempts $f'(4) = 4 \times 2 - 2 - \frac{8}{3 \times 4^2}$.

If you do not see embedded values

1) award if the candidate has two of the three terms correct. Eg two of $8-2-$ "0.166"

2) award for sight of $\frac{35}{6}$ or awrt 5.83 if the candidate just offers a value

dM1 Attempts perpendicular gradient using the correct rule. The M1 must have been awarded

M1 Attempts to find the equation of the normal using (4,1) and a **changed** gradient.

If the candidate differentiates $f'(x)$ they can be awarded this mark

Condone one sign slip when using the (4,1)

A1 $6x + 35y - 59 = 0$

Accept any $\pm k(6x + 35y - 59) = 0$ where $k \in \mathbb{N}$

(b)

M1 Raises any correct index by one $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}, 2 \rightarrow 2x, \frac{1}{x^2} \rightarrow \frac{1}{x}$ The indices must be processed

A1 Any two terms correct (may be un-simplified) with or without +c

Do not allow the indices or the coefficients to be left un processed Eg $4x^{\frac{1}{2}} \rightarrow \frac{4}{1+\frac{1}{2}} x^{\frac{1}{2}+1}$

A1 All three terms correct and simplified with or without +c. Condone spurious notation.

Look for $\frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x}$ or exact simplified equivalent such as $\frac{8x^{\frac{3}{2}}}{3} - 2x + \frac{8}{3}x^{-1}$

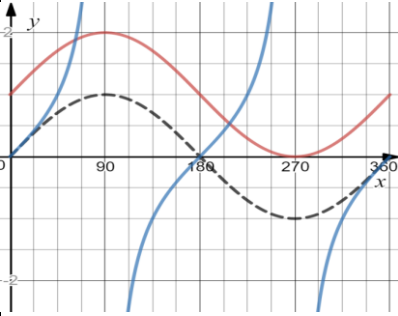
This may be implied by a final line where c has been found.

dM1 Substitute $x = 4, y = 1$ into their $f(x)$ containing +c to obtain c

A1 $f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x} - 13$ or exact simplified equivalent such as $f(x) = \frac{8}{3}x\sqrt{x} - 2x + \frac{8}{3}x^{-1} - 13$

Condone $f(x) = y$ or allow if there is no left- hand side at all.

Is w after a correct answer.

Question Number	Scheme	Marks
9. (a)	$(270^\circ, -4)$	B1 B1 (2)
(b)	 For $y = 1 + \sin \theta$ $y = \tan \theta$	B1 B1 (2)
(c)	(i) $6 \times 2 = 12$ (ii) 11	M1 A1 B1 ft (3) (7 marks)

(a)

B1 Either coordinate correct. Look for either 270° or -4 in the correct position within (,).

Alternatively look for either $x = 270$ or $y = -4$ Condone $\frac{3\pi}{2} = 270^\circ$

Do not accept multiple answers unless one point is chosen or it is clearly part of their thought process. There is no need for the degrees symbol. Condone swapped coordinates, ie $(-4, 270)$ for this mark

B1 For correct coordinates.

$(270^\circ, -4)$ with or without degrees symbol. Condone $x = 270^\circ, y = -4$

(b) These may appear on Figure 3 rather than Diagram 1

B1 For $y = 1 + \sin \theta$ Score for a curve passing through $(0,1), (90^\circ, 2), (180^\circ, 1), (270^\circ, 0), (360^\circ, 1)$ with acceptable curvature. Do not accept straight lines

B1 For $y = \tan \theta$ with acceptable curvature. Must go beyond $y = 1$ and -1

Score for the general shape of the curve rather than specific coordinates. See practice and qualification items for clarification.

First quadrant from $(0,0) \rightarrow (90^\circ, \infty)$

Second and third quadrants from $(90^\circ, -\infty) \rightarrow (270^\circ, \infty)$ passing through $(180^\circ, 0)$

Fourth quadrant from $(270^\circ, -\infty) \rightarrow (0,0)$

(c)(i) The question states hence so it is expected the results come from graphs.

If neither or only one graph is drawn then score for 12 in (i) for M1 A1 and 11 in (ii) B1

M1 For the calculation $\frac{2160}{360} = 6$ or $\frac{2160}{180} = 12$ or multiplying the number of intersections in their (b) by 6

Sight of 6 or 12 will imply this mark.

A1 12. 12 will score both marks.

(c) (ii)

B1 ft For either 11 (correct answer)

or follow through on n less than their answer to (c) (i) where n is their number of solutions in the range $180^\circ < \theta \leq 360^\circ$

Question Number	Scheme	Marks
10 (a)	$P = \left(-\frac{1}{2}, 0\right)$	B1 (1)
(b)	$f(x) = (x-4)(2x+1)^2 \Rightarrow f(x) = ax^3 + bx^2 + cx + d$ $= 4x^3 - 12x^2 - 15x - 4$ oe $f'(x) = 12x^2 - 24x - 15$	M1 A1 dM1 A1 (4)
(c)	Attempts $f'(2.5) = 12 \times 2.5^2 - 24 \times 2.5 - 15 = 0$ Finds y coordinate for $x = 2.5$ $y = -54$	M1A1 A1 (3)
(d)	$a = -\frac{1}{2}, (+) 4$	B1, B1 (2) (10 marks)

- (a)
B1 $P = \left(-\frac{1}{2}, 0\right)$ or exact equivalent. Accept coordinates written separately.
If both $(4, 0)$ and $\left(-\frac{1}{2}, 0\right)$ are given, then P must be identified as $\left(-\frac{1}{2}, 0\right)$
- (b)
M1 Attempts to multiply out to form a 4 - term cubic. The terms do not have to be collected.
Look for a multiplied-out expression that would simplify to $ax^3 + bx^2 + cx + d$
A1 $4x^3 - 12x^2 - 15x - 4$. The terms do not have to be collected for this mark.
dM1 Reduces the power by one in all terms. The indices must be processed.
Look for $ax^3 + bx^2 + cx + d \rightarrow px^2 + qx + r$ with no zero constants
A1 cso $f'(x) = 12x^2 - 24x - 15$ which must be fully simplified.
Cannot be awarded, for instance, from incorrect e.g. $f(x) = 4x^3 - 12x^2 - 15x + 4$
=

Note: May be attempted by the product rule.

M1A1: $f(x) = (x-4)(2x+1)^2 \Rightarrow f'(x) = 1 \times (2x+1)^2 + A(x-4)(2x+1)$

All 4 marks $f(x) = (x-4)(2x+1)^2 \Rightarrow f'(x) = 1 \times (2x+1)^2 + 4(x-4)(2x+1)$

Note that the terms do not need to be multiplied out under this method

(c)

M1 Attempts $f'(2.5)$ for their $f'(x)$ A1 Shows $f'(2.5) = 12 \times 2.5^2 - 24 \times 2.5 - 15 = 0$ with either embedded values shown or $f'(2.5) = 75 - 60 - 15 = 0$ For this to be scored $f'(x)$ must be correctA1 CSO Finds y coordinate for $x = 2.5 \Rightarrow$ Equation of tangent $y = -54$ but allow $k = -54$
An alternative method would be toM1 Attempts to solve their $f'(x) = 0$ A1 For $f'(x) = 0 \Rightarrow x = (-0.5), 2.5$ For this to be scored $f'(x)$ must be correctA1 CSO Finds y coordinate for $x = 2.5 \Rightarrow$ Equation of tangent $y = -54$ but allow $k = -54$

(d)

B1 For one of $-\frac{1}{2}, (+) 4$. Alternatively score for both $a = +\frac{1}{2}, -4$ Implied by $y = f\left(x - \frac{1}{2}\right)$ or $y = f(x + 4)$ for this mark onlyB1 For both $a = -\frac{1}{2}, (+) 4$ and no others. Cannot be $x = \dots$ but allow just the values $-\frac{1}{2}, (+) 4$

